SEMESTER – I

UNIT – I

Shares

(1) A company's capital is made up of 3,00,000 preference shares with 10% dividend and 7,00,000 equity shares. Both types of shares have par value of Rs. 100 each. In a particular year, the company had a profit of Rs. 90,00,000 from which Rs. 75,000 was kept in reserve and the remaining was distributed to shareholders. Find the rate of dividend received by the equity shareholders?

Solution:

$$\text{Total dividend paid to the shareholders} = \text{Total profit} - \text{Reserve fund}$$

$$= 90{,}00{,}000 - 75{,}000$$

$$= 89{,}25{,}000$$

$$\text{Dividend paid to the preference shareholders} = \frac{\text{Rate of dividend}}{100} \times \text{FV of 1 share} \times \text{No. of Shares}$$

$$= \frac{10}{100} \times 100 \times 3{,}00{,}000$$

$$= 30{,}00{,}000$$

$$\therefore \text{Dividend paid to the equity shareholders} = \text{Total dividend} - \text{dividend paid to the pref. shareholders}$$

$$= 89{,}25{,}000 - 30{,}00{,}000$$

$$= 59{,}25{,}000$$

$$\text{Dividend paid} = \frac{\text{Rate of dividend}}{100} \times \text{FV of 1 share} \times \text{No. of Shares}$$

$$59{,}25{,}000 = \frac{\text{Rate of dividend}}{100} \times 100 \times 7{,}00{,}000$$

$$\therefore \text{Rate of dividend} = \frac{5925000}{700000} = 8.46\%$$

$\therefore$ The rate of dividend paid to the equity shareholders is 8.46%.

(2) Mr. Pinto invested Rs. 50,000 in equity shares of nominal value Rs. 100 each at the market price of Rs. 125 each. The company declared an annual dividend of 12%. Find dividend amount and rate of return.

Solution:

$$\text{No. of shares purchased} = \frac{\text{Investment Amount}}{\text{Market Value of 1 share}}$$

$$= \frac{50000}{125}$$

$$= 400$$

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{matrix}\text{FV of}\\ \text{1 share}\end{matrix} \times \begin{matrix}\text{No. of}\\ \text{Shares}\end{matrix}$$

$$= \frac{12}{100} \times 100 \times 400$$

$$= 4800 \text{ Rs.}$$

$$\text{Rate of return} = \frac{\text{Total Gain}}{\text{Total Investment}} \times 100$$

$$= \frac{4800}{50000} \times 100$$

$$= 9.6\%$$

(3) Mr. Joshi invested in shares of nominal value Rs. 100 each. At 8% rate, he received a total dividend of Rs. 1,920. How many shares did he purchase?

Solution:

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{matrix}\text{FV of} \\ \text{1 share}\end{matrix} \times \begin{matrix}\text{No. of} \\ \text{Shares}\end{matrix}$$

$$1920 = \frac{8}{100} \times 100 \times \text{No. of Shares}$$

$$\therefore \quad \text{No. of Shares} = \frac{1920}{8}$$

$$= 240$$

(4) Find the face value of a share if an investment of Rs. 20,000 put into purchase 8% shares quoted at Rs. 40 each, earned a total dividend of Rs. 1,000.

Solution:

$$\text{No. of shares purchased} = \frac{\text{Investment Amount}}{\text{Market Value of 1 share}}$$

$$= \frac{20000}{40}$$

$$= 500$$

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{array}{c}\text{FV of}\\ \text{1 share}\end{array} \times \begin{array}{c}\text{No. of}\\ \text{Shares}\end{array}$$

$$1000 = \frac{8}{100} \times \begin{array}{c}\text{FV of}\\ \text{1 share}\end{array} \times 500$$

$$\text{FV of 1 Share} = \frac{1000}{40}$$

$$= 25 \text{ Rs.}$$

(5) Mr. Nitin invested Rs. 21,000 in Rs. 100 shares of a company at a rate of Rs. 150 per share. He received 7.5% dividend on these shares. Mr. Jatin invested Rs. 18,000 in Rs. 10 shares of some company at Rs. 12 per share and received 7% dividend from the company. Find which investment is more profitable.

Solution:

(1) For Nitin:

$$\text{No. of shares purchased} = \frac{\text{Investment Amount}}{\text{Market Value of 1 share}}$$

$$= \frac{21000}{150} = 140$$

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{matrix}\text{FV of}\\ \text{1 share}\end{matrix} \times \begin{matrix}\text{No. of}\\ \text{Shares}\end{matrix}$$

$$= \frac{7.5}{100} \times 100 \times 140$$

$$= 1050$$

$$\text{Rate of return} = \frac{\text{Total Gain}}{\text{Total Investment}} \times 100$$

$$= \frac{1050}{21000} \times 100 = 5\%.$$

(2) For Jatin:

$$\text{No. of shares purchased} = \frac{\text{Investment Amount}}{\text{Market Value of 1 share}}$$

$$= \frac{18000}{12} = 1500$$

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{matrix}\text{FV of}\\ \text{1 share}\end{matrix} \times \begin{matrix}\text{No. of}\\ \text{Shares}\end{matrix}$$

$$= \frac{7}{100} \times 10 \times 1500$$

$$= 1050$$

$$\text{Rate of return} = \frac{\text{Total Gain}}{\text{Total Investment}} \times 100$$

$$= \frac{1050}{18000} \times 100 = 5.83\%$$

$\therefore$ 2^{nd} investment is more profitable.

(6) Ms. Simran invested Rs. 90,180 in purchasing Rs. 100 shares at Rs. 180. The brokerage was 0.2%. Find the number of shares purchased.

Solution:

$$\begin{aligned}
\text{Cost of 1 share} &= \text{MV} + \text{Brokerage} \\
&= 180 + \frac{0.2}{100} \times 180 \\
&= 180.36 \\
\text{No. of shares purchased} &= \frac{\text{Investment Amount}}{\text{Cost of 1 share}} \\
&= \frac{90180}{180.36} \\
&= 500
\end{aligned}$$

(7) Ms. Zeel sold some shares at a market price of Rs. 250 per share with 0.3% brokerage. She got Rs. 99,700 in this transaction. Find the number of shares she sold.

Solution:

Amount received on 1 share = MV of 1 Share – Brokerage

$$= 250 - \frac{0.3}{100} \times 250$$

$$= 249.25$$

Total amount received = Amount received on 1 share × No. of Shares

99,700 = 249.25 × No. of Shares

$\therefore$ No. of Shares $= \dfrac{99700}{249.25}$

$$= 400$$

(8) Ms. Sonia got 250 shares of a company of face value Rs. 100 at a market price of Rs. 180 each. After 6 months she received dividend at 7%. After 3 months she sold the shares at a market price of Rs. 190. The brokerage was 0.2% on purchase and 0.3% on sale. Find the percentage profit.

Solution:

Cost of 1 share = MV of 1 Share + Brokerage

$$= 180 + \frac{0.2}{100} \times 180 = 180.36$$

Amt. Invested = Cost of 1 share × No. of Shares

$$= 180.36 \times 250 = 45{,}090$$

$$\text{Total Dividend Amount} = \frac{\text{Rate of dividend}}{100} \times \begin{matrix}\text{FV of}\\ \text{1 share}\end{matrix} \times \begin{matrix}\text{No. of}\\ \text{Shares}\end{matrix}$$

$$= \frac{7}{100} \times 100 \times 250 = 1750$$

Amt. received on 1 share after sale = MV of 1 share – Brokerage

$$= 190 - \frac{0.3}{100} \times 190 = 189.43$$

Total amount received after sale = Amt. received on 1 share × No. of Shares

$$= 189.43 \times 250 = 47357.5$$

$$\text{Total Profit} = \begin{matrix}\text{Total amount}\\ \text{received after sale}\end{matrix} + \begin{matrix}\text{Total dividend}\\ \text{amount}\end{matrix} - \begin{matrix}\text{Investment}\\ \text{amount}\end{matrix}$$

$$= 47357.5 + 1750 - 45090 = 4017.5$$

$$\text{Profit percentage} = \frac{\text{Total Profit}}{\text{Total Investment}} \times 100$$

$$= \frac{4017.5}{45090} \times 100 = 8.91\%$$

(9) A person invested Rs. 48,000 to buy 200 shares of a company at Rs. 240. Later on the company declared bonus shares to its existing shareholders in the ration 1:5. Post-bonus, the investor sold shares at the market price which was down to Rs. 210. Find his percentage gain or loss.

Solution:

$$\text{No. of Shares purchased} = \frac{\text{Investment Amount}}{\text{MV of 1 Share}} = \frac{48000}{240} = 200$$

$$\text{No. of bonus shares} = 200 \times \frac{1}{5} = 40$$

$$\therefore \quad \text{Total no. of shares} = 200 + 40 = 240$$

$$\text{Amt. received after sale} = \text{MV of 1 share} \times \text{No. of share} = 210 \times 240 = \text{Rs. } 50{,}400$$

$$\text{Total Gain} = \text{Amount received} - \text{Amount Invested} = 50{,}400 - 48{,}000 = 2{,}400$$

$$\text{Percentage Gain} = \frac{\text{Total Gain}}{\text{Total Investment}} \times 100 = \frac{2400}{48000} \times 100 = 5\%$$

Mutual Funds

(1) Mr. Amit invested Rs. 25,000 in a mutual fund at an NAV of Rs. 25.25 and redeemed all the units after 3 months when the NAV was Rs. 27.76. If there is no entry load and exit load, find his total gain and the rate of return.

Solution:

$$\text{No. of units purchased} = \frac{\text{Amount Invested}}{\text{Purchase NAV}}$$
$$= \frac{25000}{25.25}$$
$$= 990.099$$

$$\text{Amount received after sale} = \text{Amt. received on sale of 1 unit} \times \text{No. of units}$$
$$= 27.76 \times 990.099$$
$$= 27{,}485.15$$

$$\text{Total Gain} = \text{Amount received} - \text{Amount Invested}$$
$$= 27{,}485.15 - 25{,}000$$
$$= 2{,}485.15$$

$$\text{Rate of return} = \frac{\text{Total Gain}}{\text{Total Investment}} \times 100$$
$$= \frac{2485.15}{25000} \times 100$$
$$= 9.94\%$$

(2) Ms. Jasmine invested Rs. 39,877.5 in a mutual fund when NAV was Rs. 156. He sold all the units when the NAV reached Rs. 175. If the entry load is 2.25% and exit load is 1%, find his gain in the transaction.

Solution:

Cost of 1 Unit = Purchase NAV + Entry Load

$$= 156 + \frac{2.25}{100} \times 156$$

$$= 159.51$$

No. of units purchased $= \dfrac{\text{Amount Invested}}{\text{Cost of 1 share}}$

$$= \frac{39877.5}{159.51}$$

$$= 250$$

Amount received on sale of 1 Unit = Redemption NAV – Exit Load

$$= 175 - \frac{1}{100} \times 175$$

$$= 173.25$$

Total amount received on sale $= \dfrac{\text{Amount received on}}{\text{sale of 1 Unit}} \times$ No. of units

$$= 173.25 \times 250$$

$$= 43,312.50$$

$\therefore$ Total Gain = Total amount received – Amount Invested

$$= 43,312.50 - 39877.5$$

$$= 3435$$

(3) Mr. Lalit invested Rs. 85,272 in a mutual fund at an NAV of Rs. 52.25. Due to entry load, the value of his units on that day was Rs. 83,600. Find the number of units and the entry load.

Solution:

$$\text{Value of Investment} = \text{NAV} \times \text{No. of units}$$

$$83{,}600 = 52.25 \times \text{No. of units}$$

$$\therefore \quad \text{No. of units} = \frac{83600}{52.25} = 1600$$

$$\text{Total entry load paid} = \text{Investment Amount} - \text{Value of Investment}$$

$$= 85{,}272 - 83{,}600$$

$$= 1672$$

$$\therefore \quad \text{Entry load on 1 unit} = \frac{\text{Total entry load}}{\text{No. of units}}$$

$$= \frac{1672}{1600}$$

$$= 1.045$$

$$\text{Entry load on 1 unit} = \text{Purchase NAV} \times \frac{\text{Rate of entry load}}{100}$$

$$1.045 = 52.25 \times \frac{\text{Rate of entry load}}{100}$$

$$\therefore \quad \text{Rate of entry load} = \frac{1.045 \times 100}{52.25}$$

$$= 2\%$$

(4) Mr. Jain purchased 360 units of a mutual fund and redeemed all of them after 5 months when the NAV rose to Rs. 175. The entry and exit load were 2.25% and 1% respectively. His total gain was Rs. 3,474. Find the purchase NAV.

Solution:

Let purchase NAV = Rs. x

$\therefore$ Purchase price of 1 unit = Purchase NAV + Entry Load

$$= x + \frac{2.25}{100} \times x$$

$$= 1.0225x$$

$\therefore$ Amount received after sale of 1 Unit = Redemption NAV – Exit Load

$$= 175 - \frac{1}{100} \times 175$$

$$= 173.25$$

$\therefore$ Total amount received after sale = 173.25×360

$$= 62{,}370$$

$\therefore$ Total Gain = Total amount received – Amount Invested

$$3474 = 62{,}370 - 368.10x$$

$$\therefore \quad 368.10x = 62{,}370 - 3474$$

$$\therefore \quad x = \frac{58896}{368.10}$$

$$= 160$$

$\therefore$ Purchase NAV = 160 Rs.

(5) Ms. Priya invested Rs. 39,877.5 in a mutual fund when the NAV was Rs. 312 with an entry load of 2.25%. After receiving a dividend @ Rs. 4 per unit she waited for 6 months and redeemed all the units and paid an exit load of 0.5%. The total gain was Rs. 222.5. What was the NAV at which she redeemed the units?

Solution:

Purchase price of 1 unit = Purchase NAV + Entry Load

$$= 312 + \frac{2.25}{100} \times 312$$

$$= 319.02$$

$$\text{No. of units purchased} = \frac{\text{Amount Invested}}{\text{Purchased price of 1 unit}}$$

$$= \frac{39877.5}{319.02}$$

$$= 125$$

Total dividend amount = dividend on 1 unit × No. of units

$$= 4 \times 125$$

$$= 500$$

Let redemption NAV = Rs. x.

Amount received after sale of 1 Unit = Redemption NAV – Exit Load

$$= x - \frac{0.5}{100}x$$

$$= \frac{99.5}{100}x = 0.995x$$

Total amount received after sale $= 0.995x \times 125$

$$= 124.375x$$

Total Gain = Amount received – Amount Invested

$$222.5 = 124.375x + 500 - 39877.5$$

$$\therefore \quad 124.375x = 39877.5 - 500 + 222.5$$

$$\therefore \quad x = \frac{39600}{124.375} = 318.39$$

$\therefore$ Redemption NAV = Rs. 318.39

(6) Mr. Deepak invested Rs. 50,000 in a mutual fund in the dividend reinvestment option, on 13/07/18, when the NAV was Rs. 186.4 and the entry load was 2.25%. The fund declared a dividend @ Rs. 4 per unit on 24/12/18 and the ex-dividend NAV was Rs. 188.6. Find the total number of units (upto 3 decimal places) after the dividend is reinvested.

Solution:

$$\text{Purchase price of 1 unit} = \text{Purchase NAV + Entry Load}$$

$$= 186.4 + \frac{2.25}{100} \times 186.4$$

$$= 190.594$$

$$\text{No. of units purchased} = \frac{\text{Investment Amount}}{\text{Purchased price of 1 unit}}$$

$$= \frac{50000}{190.594}$$

$$= 262.338$$

$$\text{Total dividend} = \text{Dividend on 1 unit} \times \text{No. of units}$$

$$= 4 \times 262.338 = 1049.352$$

$$\text{No. of units reinvested} = \frac{\text{Total Dividend}}{\text{Ex-dividend NAV}}$$

$$= \frac{1049.352}{188.6}$$

$$= 5.564$$

$$\therefore \quad \text{Total no. of units} = 262.338 + 5.564 = 267.902$$

(7) Ms. Reema invested Rs. 5000 on 5th of every month for 5 months in a SIP of a mutual fund. The NAVs on these dates were Rs. 36.82, Rs. 35.66, Rs. 38.69, Rs. 39.12 and Rs. 39.78. Find the average acquisition cost per unit. (The number of units are rounded up to 3 decimal places)

Solution:

Month	Investment Amount	NAV	No. of units
1	5,000	36.82	135.796
2	5,000	35.66	140.213
3	5,000	38.69	129.232
4	5,000	39.12	127.811
5	5,000	39.78	125.691
	Total Investment = 25,000		**Total no. of units = 658.743**

$$\therefore \text{ Average Acquisition Cost } = \frac{\text{Total Investment}}{\text{Total no. of units}}$$

$$= \frac{25000}{658.743}$$

$$= 37.95$$

(8) Mr. Deshpande joined the SIP scheme for a mutual fund under which he would invest Rs. 15,000 each for 5 months. If the NAVs for each month are Rs. 43.6, Rs. 45.2, Rs. 44.5, Rs. 43.2 and Rs. 46.8 respectively. The entry load was 2.25% throughout for these months. Find his average acquisitions cost per unit. (The number of units are rounded upto 3 decimal places).

Solution:

Month	Investment Amount	NAV	Purchase Price	No. of units
1	15,000	43.6	44.58	336.474
2	15,000	45.2	46.22	324.535
3	15,000	44.5	45.50	329.670
4	15,000	43.2	44.17	339.597
5	15,000	46.8	47.85	313.480

Total Investment = 75,000 **Total no. of units = 1643.756**

$$\therefore \text{ Average Acquisition Cost } = \frac{\text{Total Investment}}{\text{Total no. of units}}$$

$$= \frac{75000}{1643.756}$$

$$= 45.63$$

UNIT – II

Permutations and Combinations

(1) Find n, for each of the following:

(a) ${}^{n}P_{4} = {}^{n}P_{5}$

(b) ${}^{n}P_{3} = 4\,({}^{n}P_{2})$

(c) ${}^{2n}P_{3} = 20\,({}^{n}P_{2})$

(d) $3\,({}^{n}P_{4}) = 2\,({}^{n+1}P_{4})$

Solution:

Formula: $${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

(i)
$$
\begin{aligned}
{}^{n}P_{4} &= {}^{n}P_{5}\\
\frac{n!}{(n-4)!} &= \frac{n!}{(n-5)!}\\
\frac{n!}{(n-4)\,(n-5)!} &= \frac{n!}{(n-5)!}\\
\frac{1}{(n-4)} &= 1\\
1 &= n-4\\
n &= 5
\end{aligned}
$$

(ii)
$$
\begin{aligned}
{}^{n}P_{3} &= 4\left({}^{n}P_{2}\right)\\
\frac{n!}{(n-3)!} &= 4\frac{n!}{(n-2)!}\\
\frac{n!}{(n-3)!} &= 4\frac{n!}{(n-2)\,(n-3)!}\\
1 &= \frac{4}{n-2}\\
n-2 &= 4\\
n &= 6
\end{aligned}
$$

(iii)
$$
\begin{aligned}
{}^{2n}P_{3} &= 20\left({}^{n}P_{2}\right)\\
\frac{2n!}{(2n-3)!} &= 20\,\frac{n!}{(n-2)!}\\
\frac{2n(2n-1)\,(2n-2)\,(2n-3)!}{(2n-3)!} &= 20\times\frac{n(n-1)\,(n-2)!}{(n-2)!}\\
2n\,(2n-1)\,(2n-2) &= 20\times n(n-1)\\
4n\,(2n-1)\,(n-1) &= 20\times n(n-1)\\
2n-1 &= \frac{20}{4}\\
2n-1 &= 5\\
2n &= 6
\end{aligned}
$$

$$n = 3$$

(iv)

$$3\left({}^{n}P_4\right) = 2\left({}^{n+1}P_4\right)$$

$$3\,\frac{n!}{(n-4)!} = 2\,\frac{(n+1)!}{(n+1-4)!}$$

$$3\,\frac{n!}{(n-4)!} = \frac{2(n+1)n!}{(n-3)!}$$

$$3\,\frac{n!}{(n-4)!} = \frac{2(n+1)n!}{(n-3)\,(n-4)!}$$

$$3 = \frac{2n+2}{n-3}$$

$$3(n-3) = 2n+2$$

$$3n-9 = 2n+2 \Rightarrow n = 11$$

(2) How many three digit numbers can be formed out of the digits 1, 2, 3, 4, 5, 6, 7, if no digit is repeated in any number? How many of these will be greater than 350?

Solution:

(i) * Three digit number without repetition of digits.

** Digits to be used: 1,2,3,4,5,6,7

Hundred's Place   Ten's Place   Unit's Place

Starting from hundred's place, we have 7 choices to fill hundred's place. After filing up hundred's place, we have 6 choices to fill ten's place as repetition is not allowed. After filing up ten's place, we have 5 choices to fill unit's place.

$\therefore$ By product rule, all the three places can be filled in $7 \times 6 \times 5 = 210$ ways.

$\therefore$ Three are 210 three digit numbers formed out of the digits 1,2,3,4,5,6 and 7 if no digits is repeated.

(ii) Since the number must be greater than 350, the hundred's place cannot have digits 1 and 2. If the hundred's place has the digits 4,5,6 or 7 then the number will clearly be greater than 350 but if the hundred's place has digit 3, then we need to ensure that the ten's place has at least 5. thus we have to consider two separate cases.

Case I: Suppose hundred's place has digit 4,5,6 or 7.

So, in this case hundred's place can be filled in 4 ways, ten's place can be filled in 6 ways and unit's place can be filled in 5 ways.

$\therefore$ By product rule, all the three places can be filled in $4 \times 6 \times 5 = 120$ ways.

Case II: Suppose hundred's place has digit 3.

So, in this case hundred's place can be filled in 1 ways, ten's place can be filled by using digits 5,6 or 7 (i.e.) in 3 ways and unit's place can be filled in 5 ways.

$\therefore$ By product rule, all the three places can be filled in $1 \times 3 \times 5 = 15$ ways.

$\therefore$ By sum rule, there are $120 + 15 = 135$ three digit numbers formed out of the digits 1,2,3,5,6 and 7, greater than 350 if no digit is repeated.

(3) How many different words beginning and ending with a vowel can be formed out of the letters of the word 'COMPUTER'?

Solution: Arrangement Problem:

There are 3 vowels in the word COMPUTER.

We have to find number of arrangements begin and end with a vowel.

So, we have to arrange first two vowels out of three on first and last position, which can be done in $^3P_2 = \frac{3!}{(3-2)!} = 6$ ways.

After making arrangement for first and last position, We left with 6 letters and 6 position. This arrangement can be done in $^6P_6 = 6! = 720$ ways.

Thus by product rule, there are $6 \times 720 = 4320$ different words beginning and ending with vowel can be formed out of the letters of the word 'COMPUTER'.

(4) Six boys and four girls are made to stand in a line.

(a) How many different arrangements can be made so that, no two girls are together?

(b) How many different arrangements can be made so that, all four girls are always together?

Solution:

(a) We have to arrange six boys and four girls in a line so that, no two girls are together. To do so, first make 6 boys to stand in a line, which can be done in ${}^6P_6$ = 720 ways.

$\square\ B_1\ \square\ B_2\ \square\ B_3\ \square\ B_4\ \square\ B_5\ \square\ B_6\ \square$

As no two girls are to sit together, they have 5 places between the boys and the 2 places at the two ends (i.e.) totally 7 places available.

So, we have to arrange 4 girls on this 7 available positions, which can be done in ${}^7P_4 = \frac{7!}{3!}$ = 840 ways.

∴ By product rule, there are 720 × 840 = 6,04,800 different arrangements.

(b) We have to arrange 6 boys and 4 girls in a line so that, all 4 girls are always together.

To do so, consider the 4 girls group as 1 since we can't break this 4 girls group as per the requirement. so we have 6 boys + 1 (4-girls) groups to arrange, which can be done in ${}^7P_7$ = 5040 ways. After this arrangement note that girls group can be rearranged in themselves in ${}^4P_4$ = 24 ways.

∴ by product rule, there are 5040 × 24 = 1,20,960 different arrangements.

(5) In how many ways can 5 Statistics books, 4 Accounts books and 2 Economics books be arranged on a shelf, if the books on the same subject are to be together?

Solution: Arrangement Problem:

We have to arrange books so that the books on the same subject are to be together.

To do so, consider the books on the same subject as 1 lot. So we have 3 such lots and this 3 lots can be arranged in ${}^{3}P_{3} = 6$ ways.

After arranging the lots, 5 statistics books can be arranged amongst themselves in ${}^{5}P_{5} = 120$ ways.

4 accounts books can be arranged in ${}^{4}P_{4} = 24$ ways and 2 economics books can be arranged in ${}^{2}P_{2} = 2$ ways.

$\therefore$ By product rule, number of arrangement = $6 \times 120 \times 24 \times 2 = 34{,}560$.

(6) How many different words can be formed out of the letters of the word 'SOCIOLOGICAL'?

Solution: Arrangement with Repetition Problem:

Formula:

$$\text{Number of arrangements} = \frac{n!}{n_1!\ n_2!\ n_3!\ \ldots\ n_k!}$$

Here, n = total number of positions = 12

n_1 = number of times 'O' repeating = 3

n_2 = number of time 'C' repeating = 2

n_3 = number of times 'I' repeating = 2

n_4 = number of times 'L' repeating = 2

$$\therefore \text{ number of arrangements} = \frac{12!}{3!\ 2!\ 2!\ 2!}$$

$$= 99{,}79{,}200$$

(7) In how many ways can a team of 11 cricketers be chosen from 10 batsmen and 6 bowlers to give a majority of batsman, if at least 4 bowlers are to be included.

Solution: Selection Problem:

We have to select a team of 11 with majority of batsman from 10 batsman and 6 bowlers the number of batsmen must be at least 6.

We also have to include at least 4 bowlers.

This can be done as.

(i) 4 bowlers and 7 batsmen or

(ii) 5 bowlers and 6 batsmen

Case (i) 4 bowlers and 7 batsmen in team.

Here we have to select 4 bowlers from 6, which can be done in

$$^{6}C_{4} = {}^{6}C_{2} = \frac{6 \times 5}{2} = 15 \text{ ways.}$$

And 7 batsmen from 10, which can be done in

$$^{10}C_{7} = {}^{10}C_{3} = \frac{10 \times 9 \times 8}{3!} = 120 \text{ ways.}$$

$\therefore$ by product rule,

Number of teams with 4 bowlers and 7 batsmen = 15 $\times$ 120 = 1800.

Case (ii) 5 bowlers and 6 batsmen in team.

Here we have to select 5 bowlers from 6 which can be done in

$$^{6}C_{5} = {}^{6}C_{1} = 6 \text{ ways.}$$

And 6 batsmen from 10, which can be done in

$$^{10}C_{6} = {}^{10}C_{4} = \frac{10 \times 9 \times 8 \times 7}{4!} = 210 \text{ ways.}$$

$\therefore$ by product rule,

Number of team with 5 bowlers and 6 batsmen = 6 $\times$ 210 = 1260.

$\therefore$ by sum rule,

Number of ways to select a team = 1800 + 1260 = 3060.

(8) A committee of 4 persons is to be formed from 10 men and 6 women. Find the total number of ways if committee consist of (i) All men (ii) 3 men and 1 woman (iii) at least one woman.

Solution: Selection Problem:

(i) All men.

Here we have to select 4 men from 10, which can be done in $^{10}C_4 = \dfrac{10 \times 9 \times 8 \times 7}{4!} = 210$ ways.

$\therefore$ Committee with all 4 members men can be formed in 210 ways.

(ii) 3 men and 1 women.

Here we have to select 3 men from 10, which can be done in $^{10}C_3 = \dfrac{10 \times 9 \times 8}{3!} = 120$ ways and 1 woman from 6, which can be done in $^{6}C_1 = 6$ ways.

By product rule,

Committee with 3 men and 1 women can be formed in $120 \times 6 = 720$ ways.

(iii) At least one woman

Compliment Method: Complement of at least one woman is zero woman.

Step (I): committee of 4 persons can be formed from 11 person in $^{11}C_4 = \dfrac{11 \times 10 \times 9 \times 8}{4!} = 330$ ways.

Step (II): committee with all members men can be formed in $^{10}C_4 = 210$ ways.

Step (III): committee with at least one woman can be formed in $330 - 210 = 120$ ways.

(9) A committee of 3 people is to be selected from a group of 8 people, which includes 4 married couples. If the committee cannot contain more than one member of any married couple, how many 3 person committees are possible?

Solution: Selection Problem:

We have to form a committee of 3 people from 8, which includes 4 married couples. Since the committee cannot contain more than one member of any married couple, we shall first select 3 couples from 4 and form each selected couple, select one person.

Couples can be selected in ${}^4C_3 = {}^4C_1 = 4$ ways.

From each selected couple. One person can be selected in ${}^2C_1 = 2$ ways.

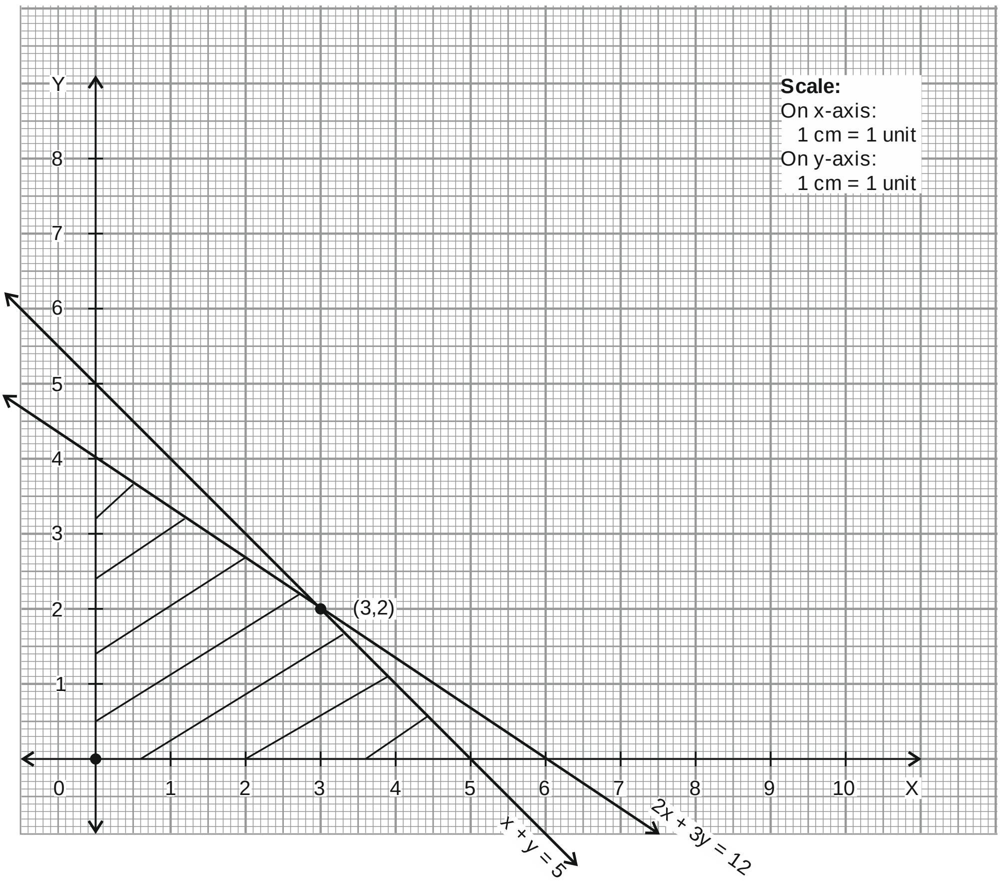
$\therefore$ Number of 3 person committees $= 4 \times 2 \times 2 \times 2 = 32$.

Linear Programming Problem

Solve the following L.P.P. Graphically:

(1) Maximize Z = 4x + 5y

Subject to constraints: 2x + 3y ≤ 12, x + y ≤ 5, x ≥ 0, y ≥ 0.



Solution:

Consider,

$2x + 3y = 12$			$x + y = 5$		
x	y	(x,y)	x	y	(x,y)
0	4	(0,4)	0	5	(0,5)
6	0	(6,0)	5	0	(5,0)

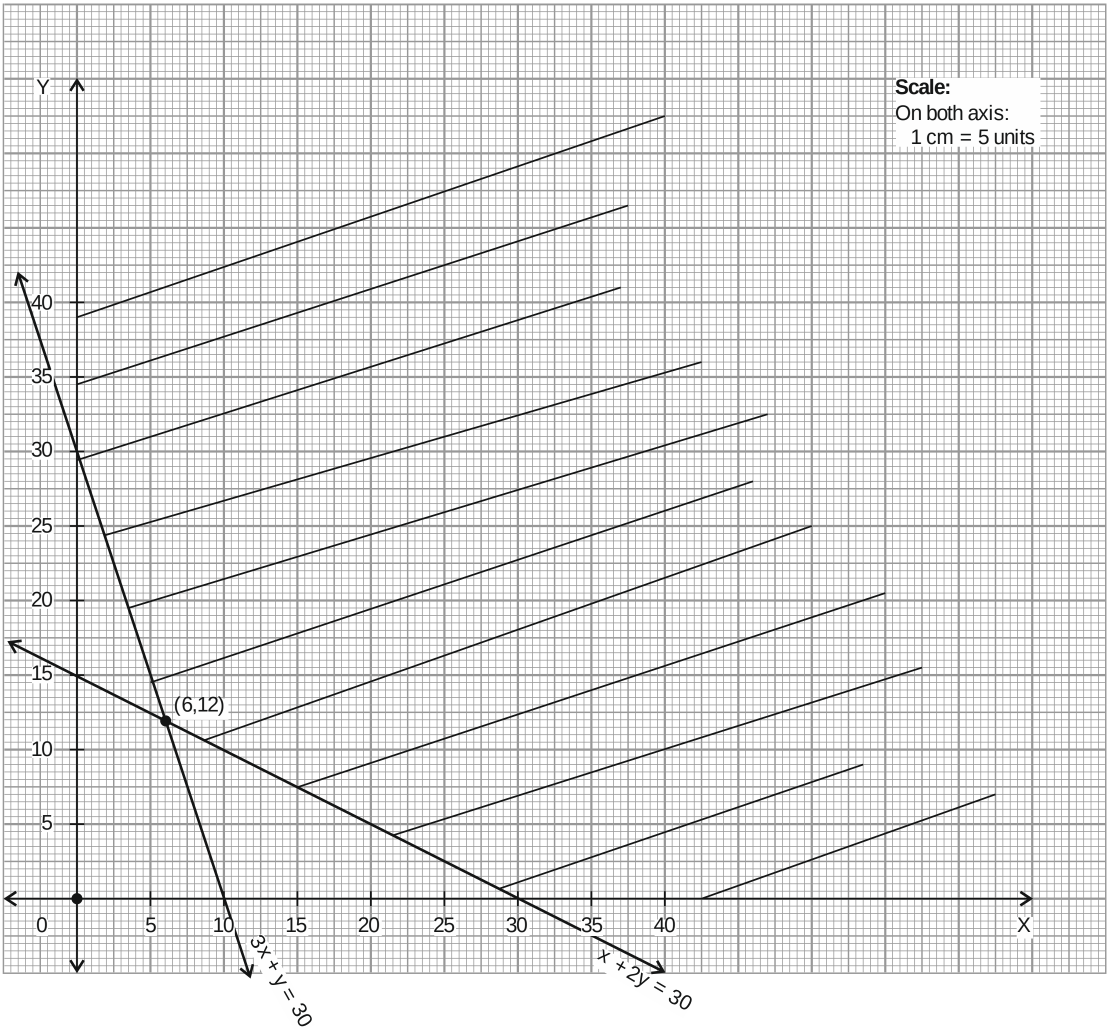
$2x + 3y \le 12$	$x + y \le 5$
Put $x = y = 0$	Put $x = y = 0$
$0 \le 12$ (True)	$0 \le 5$ (True)
Region is origin side	Region is origin side

Points	Z	$= 4x + 5y$
(0,0)	Z	= 0
(5,0)	Z	= 20
(3,2)	Z	= 12 + 10 = 22
(0,4)	Z	= 20

$\therefore$ Objective function $Z = 4x + 5y$ is maximum at (3,2) with value 22.

(2) Minimize $Z = 9x + 10y$

Subject to Constraints: $x + 2y \geq 30$, $3x + y \geq 30$, $x \geq 0$, $y \geq 0$.



Solution:

Consider,

$x + 2y = 30$			$3x + y = 30$		
x	y	(x,y)	x	y	(x,y)
0	15	(0,15)	0	30	(0,30)
30	0	(30,0)	10	0	(10,0)

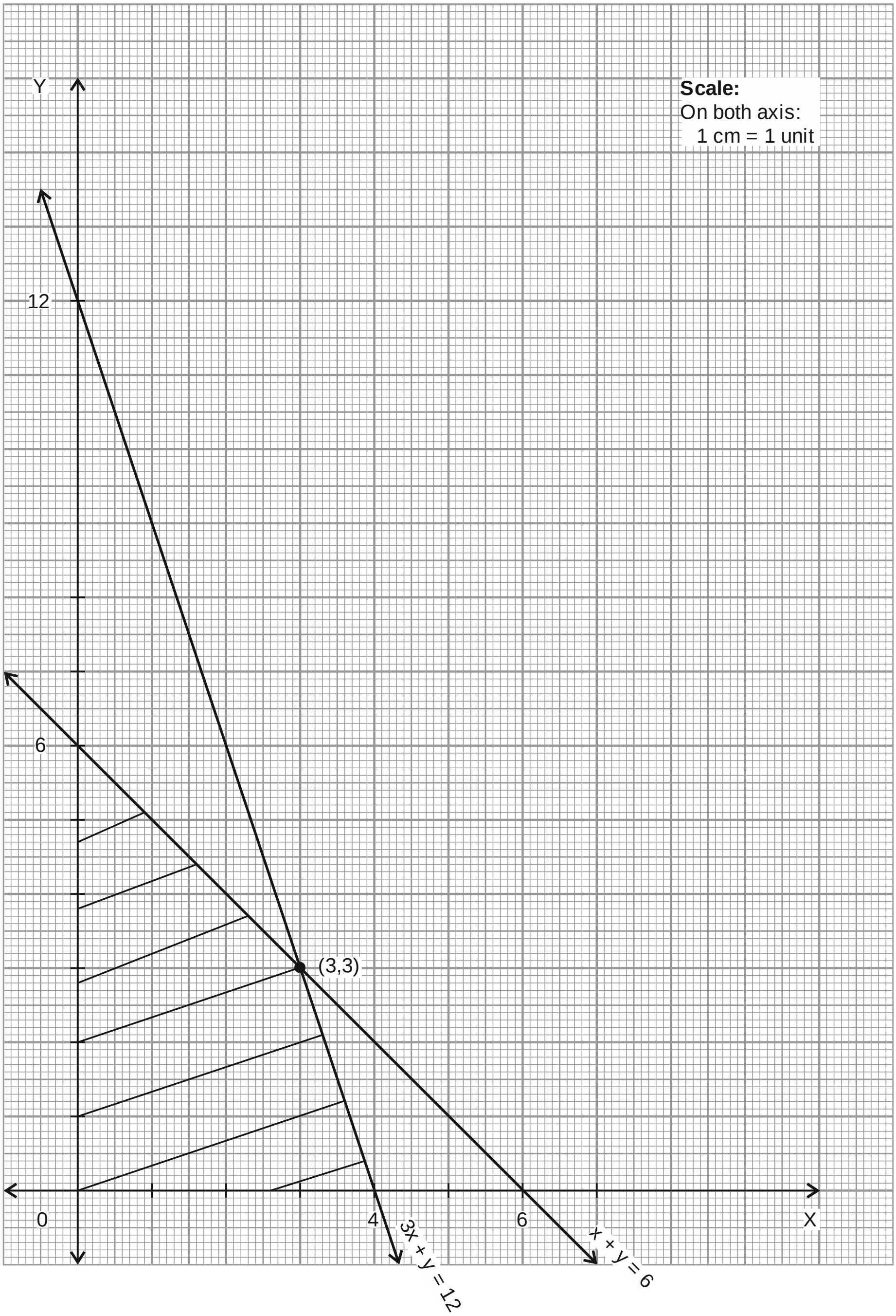
$x + 2y \geq 30$	$3x + y \geq 30$
Put x = y = 0	Put x = y = 0
$0 \geq 30$ (False)	$0 \geq 30$ (False)
Region is non-origin side	Region is non-origin side

Points	Z	= 9x + 10y
(0,30)	Z	= 300
(6,12)	Z	= 54 + 120 = 174
(30,4)	Z	= 270

$\therefore$ Objective function Z = 9x + 10y is minimum at (6,12) with value 172.

(3) Maximize $z = 2x + 3y$

Subject to Constraints: $x + y \leq 6$, $3x + y \leq 12$, $x \geq 0$, $y \geq 0$.



Solution:

Consider,

$3x + y = 12$

x	y	(x,y)
0	12	(0,12)
4	0	(4,0)

$3x + y \leq 12$

$0 \leq 12$ (True)

$\therefore$ Region is origin side

$x + y = 6$

x	y	(x,y)
0	6	(0,6)
6	0	(6,0)

$x + y \leq 6$

$0 \leq 6$ (True)

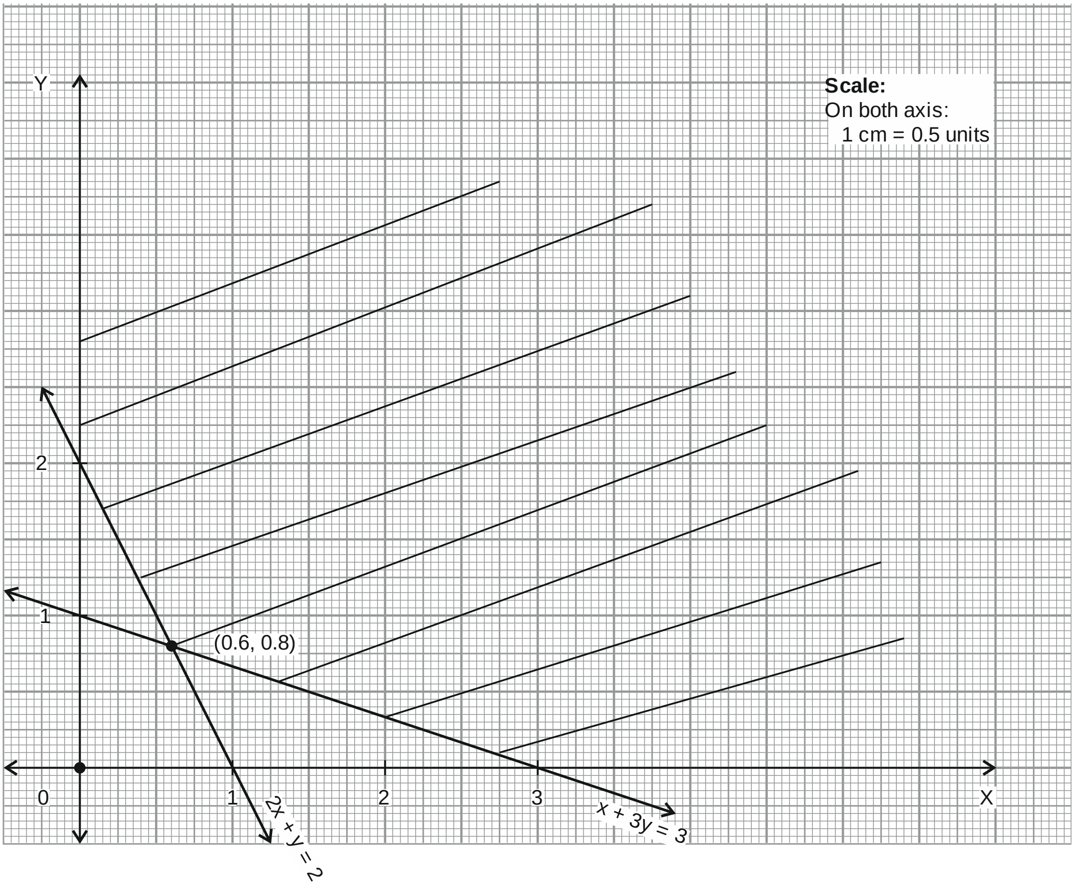
$\therefore$ Region is origin side

Points	Z	= 2x + 3y
(0,0)	Z	= 0
(4,0)	Z	= 8
(3,3)	Z	= 6 + 9 = 15
(0,6)	Z	= 18

$\therefore$ Objective function $Z = 2x + 3y$ is maximum at (0,6) with value 18.

(4) Minimize Z = 10x + 7y

Subject to Constraints: $2x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.



Solution:

Consider,

$2x + y = 2$

x	y	(x,y)
0	2	(0,2)
1	0	(1,0)

$2x + y \geq 2$

$0 \geq 2$ (False)

$\therefore$ Region is non-origin side

$x + 3y = 3$

x	y	(x,y)
0	1	(0,1)
3	0	(3,0)

$x + 3y \geq 3$

$0 \geq 3$ (False)

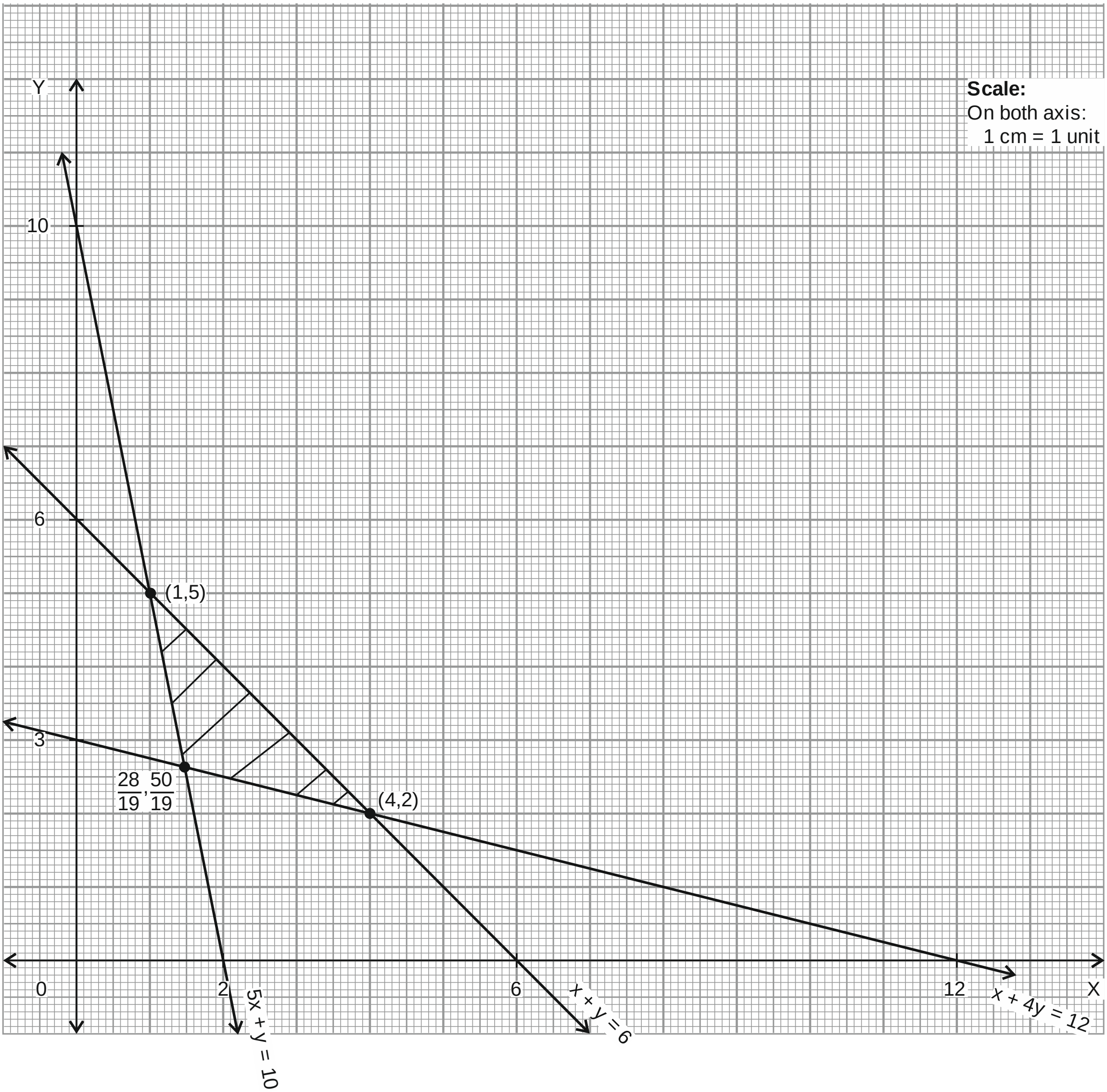
$\therefore$ Region is non-origin side

Points	Z	= 10x + 7y
(0,2)	Z	= 14
(0.6,0.8)	Z	= 6 + 5.6 = 11.6
(3,6)	Z	= 30

$\therefore$ Objective function $Z = 10x + 7y$ is minimum at (0.6,0.8) with value 11.6

(5) Minimize $Z = 80x + 40y$

Subject to Constraints: $5x + y \geq 10$, $x + y \leq 6$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$.



Solution:

Consider,

$5x + y = 10$			$x + y = 6$			$x + 4y = 12$		
x	y	(x,y)	x	y	(x,y)	x	y	(x,y)
0	10	(0,10)	0	6	(0,6)	0	3	(0,3)
2	0	(2,0)	6	0	(6,0)	12	0	(12,0)

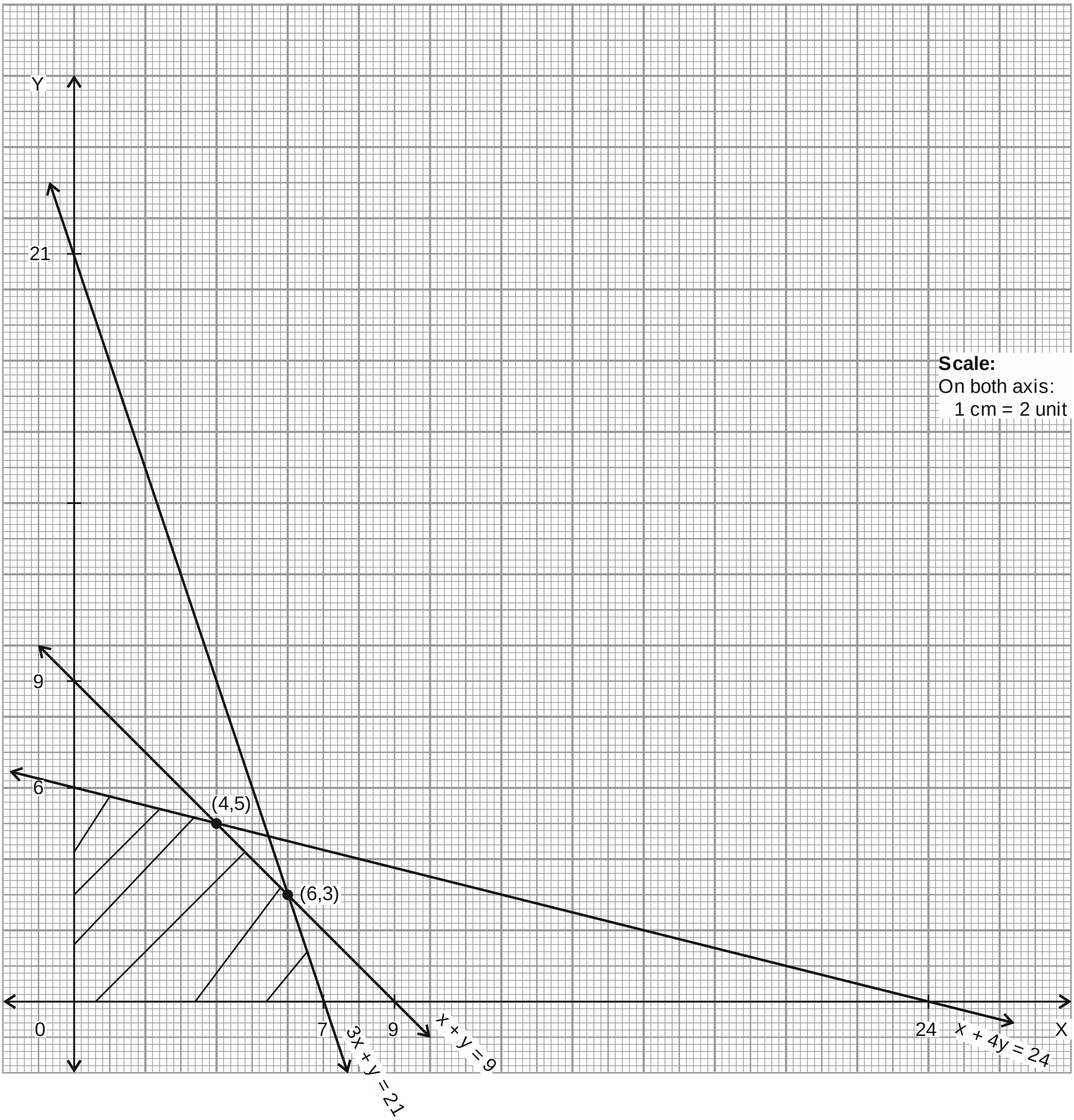
$5x + y \geq 10$	$x + y \leq 6$	$x + 4y \geq 12$
$0 \geq 10$ (False)	$0 \leq 6$ (True)	$0 \geq 12$ (False)
non-origin side	origin side	non-origin side

Points	Z	$= 80x + 40y$
(1,5)	Z	$= 80 + 200 = 280$
(4,2)	Z	$= 320 + 80 = 400$
$\left(\frac{20}{19}, \frac{50}{19}\right)$	Z	$= \frac{1600}{19} + \frac{2000}{19} = 189.47$

$\therefore$ Objective function $Z = 80x + 40y$ is minimum at $\left(\frac{20}{19}, \frac{50}{19}\right)$ with value 189.47.

(6) Maximize Z = x + y

Subject to Constraints: $x + 4y \le 20$, $3x + y \le 21$, $x + y \le 9$, $x \ge 0$, $y \ge 0$.



Solution:

Consider,

$x + 4y = 20$			$3x + y = 21$			$x + y = 9$		
x	y	(x,y)	x	y	(x,y)	x	y	(x,y)
0	5	(0,5)	0	21	(0,21)	0	9	(0,9)
20	0	(20,0)	7	0	(7,0)	9	0	(9,0)

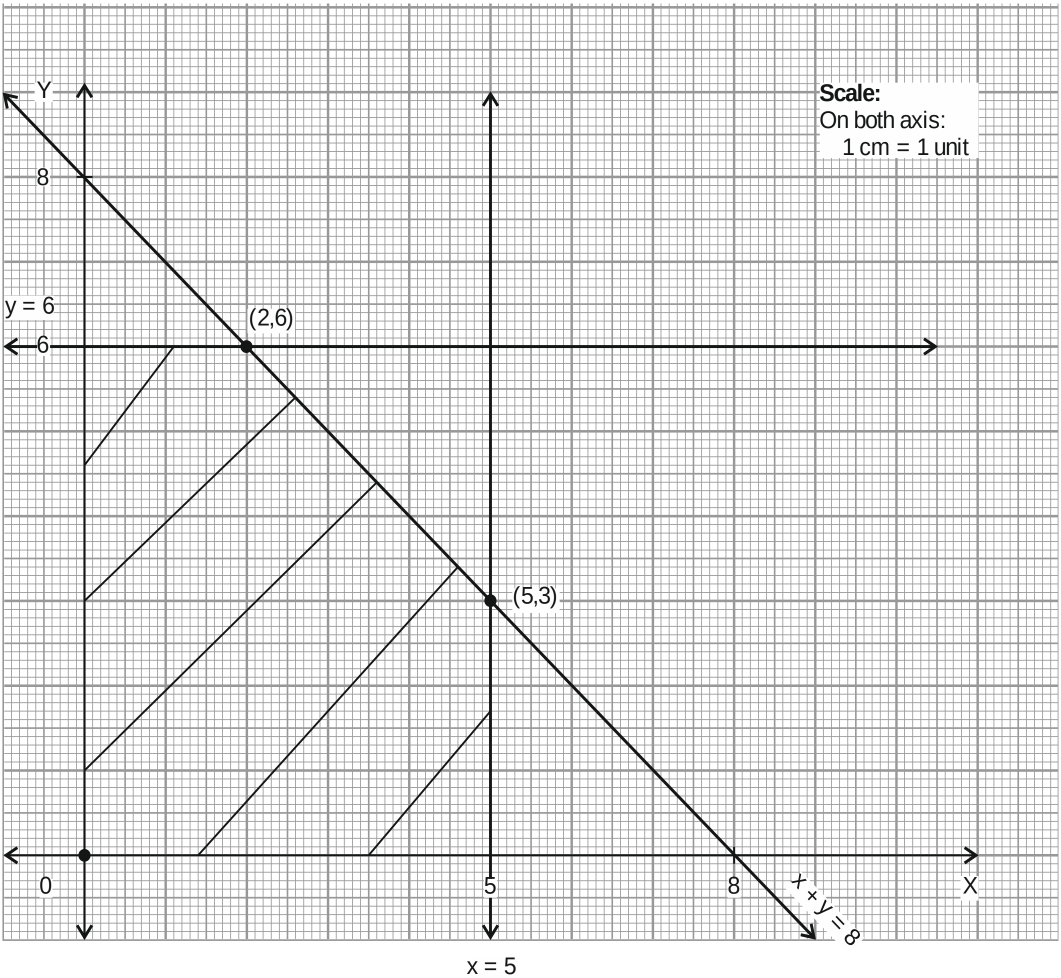
$x + 4y \le 20$	$3x + y \le 21$	$x + y \le 9$
$0 \le 20$ (True)	$0 \le 21$ (True)	$0 \le 9$ (True)
origin side	origin side	origin side

Points	Z	= x + y
(0,0)	Z	= 0
(7,0)	Z	= 7
(6,3)	Z	= 9
(4,5)	Z	= 9
(0,6)	Z	= 6

$\therefore$ Objective function $Z = x + y$ is maximum at each point on the line segment joining points (6,3) and (4,5) with value 9.

(7) Maximize Z = x + y

Subject to Constraints: $x \le 5$, $y \le 6$, $x + y \le 8$, $x \ge 0$, $y \ge 0$.



Solution:

Consider,

$x + y = 8$

x	y	(x,y)
0	8	(0,8)
8	0	(8,0)

$x + y \le 8$	$x \le 5$	$y \le 6$
Put $x = y = 0$	Put $x = 0$	Put $y = 0$
$0 \le 8$ (True)	$0 \le 5$ (True)	$0 \le 6$ (True)
∴ Region is origin side	∴ Region is origin side	∴ Region is origin side

Points	Z	= x + y
(0,0)	Z	= 0
(5,0)	Z	= 5
(5,3)	Z	= 8
(2,6)	Z	= 8
(0,6)	Z	= 6

∴ Objective function $Z = x + y$ is maximum at each point on the line segment joining points (5,3) and (2,6) with value 8.

Formulate the following L.P.P.

(1) A company manufactures two kinds of chemicals viz, A and B. Each requires three types of raw materials P, Q and R. Chemical A is produced by using 3 units of P, 5 units of Q and 2 units of R. Chemical B is produced by using 3 units of P, 2 units of Q and 6 units of R. Only 36 units of P, 50 units of Q and 60 units of R are available. The profit on chemical A is Rs. 20 and on B, it is Rs. 30 per unit. Formulate a LPP to maximize the profit.

Solution:

Let no. of units of chemical A produced = x
and no. of units of chemical B produced = y.

Raw Materials	Chemical A (x)	Chemical B (y)	Maximum Availability (≤)
P	3	3	36
Q	5	2	50
R	2	6	60
Profit	20	30	

Formulation:

Maximize, $Z = 20x + 30y$

Subject to the constraints, $3x + 3y \le 36$, $5x + 2y \le 50$,
$2x + 6y \le 60$, $x, y \ge 0$.

(2) Two types of food packets, P and Q to be mixed in the fodders at a cattle farm are available, both containing Nutrients N_1, N_2 and N_3. A cattle needs 108mg of N_1, 36mg of N_2 and 100 mg of N_3 per meal. The packet P contains 36mg of N_1, 3mg of N_2 and 20mg of N_3 and Q contains 6mg of N_1, 12mg of N_2 and 10mg of N_3. Cost of these food packets are Rs. 20 and Rs. 40 respectively. Formulate the LPP to minimize the cost.

Solution:

Let no. of packets of P purchased = x
and no. of packets of Q purchased = y.

	Food Packet		
	P	**Q**	**Minimum Requirement**
Nutrients	**(x)**	**(y)**	**(≥)**
N_1	36	6	108
N_2	3	12	36
N_3	20	10	100
Cost	20	40	

Formulation:

Minimize, $Z = 20x + 40y$

Subject to the constraints, $36x + 6y \geq 108$, $3x + 12y \geq 36$,
$20x + 10y \geq 100$, $x, y \geq 0$.

(3) A printing company prints two types of magazine A and B. The company earns Rs. 10 and Rs. 15 on magazines A and B respectively. These are processed on three machines, namely machine I, II and III. Magazine A requires 2 hours on machine I, 5 hours on machine II and 2 hours on machine III. Magazine B requires 3 hours on machine I, 2 hours on machine II and 6 hours on machine III. Machine I, II and III are available for at most 36, 50 and 60 hours per week respectively. Formulate the linear programming problem to determine weekly production of magazines A and B so that the total profit will be maximum.

Solution:

Let weekly production of magazine A = x
and weekly production of magazine B = y.

	Magazine		
	A	**B**	**Maximum Availability**
Machine	**(x)**	**(y)**	**(≤)**
I	2	3	36
II	5	2	50
III	2	6	60
Profit	10	15	60

Formulation:

Maximize, $Z = 10x + 15y$

Subject to the constraints, $2x + 3y \le 36$, $5x + 2y \le 50$,
$2x + 6y \le 60$, $x, y \ge 0$.

(4) Daily requirement of two vitamins V_1 and V_2 and the mineral M for a certain person is atleast 30 units of V_1, 60 units of V_2 but not more than 40 units of M. He meet this requirement by taking two brands of tablets A and B. Tablet A has 3 units of V_1, 4 units of V_2 and 1 unit of M. Tablet B has 1 unit of V_1, 3 units of V_2 and 2 units of M. Tablet A costs Rs. 2 and tablet B cost Re. 1. Formulate a L.P.P. to minimize his expenditure.

Solution:

Let no. of tablets of A consumed = x
and no. of tablets of B consumed = y.

	Tablet A (x)	**Tablet B (y)**	**Minimum Requirement (≥)**
Vitamin V_1	3	1	30
Vitamin V_2	4	3	60
Mineral M	1	2	Maximum (≤) 40
Cost	2	1	

Formulation:

Minimize, $Z = 2x + y$

Subject to the constraints, $3x + y \geq 30$, $4x + 3y \geq 60$,
$x + 2y \leq 40$, $x, y \geq 0$.

(5) A post office wants to hire temporary helpers during Diwali season that cannot exceed 7 employees. Experience from past tells that a man can handle 100 letters and 200 packages per day and a woman can handle 200 letters and 100 packages per day. The post master expects that daily volume of letters and packages will not be less than 900 and 900 respectively. The wages for a man are Rs. 100 and for a woman Rs. 90 per day. Frame a LPP to minimize the hiring charges.

Solution:

Let no. of men hired = x,
and no. of women hired = y.
Given that we can't hire more than 7 employees.

$$\Rightarrow x + y \le 7 \qquad \ldots\ldots\ldots \textbf{(1)}$$

Also given that a man can handle 100 letters and 200 package per day and a woman can handle 200 letters and 100 packages per day and expected daily volume of letter and packages will not be less than 900 for both.

$$\Rightarrow \left.\begin{aligned} 100x + 200y &\ge 900 \\ 200x + 100y &\ge 900 \\ x, y &\ge 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots \textbf{(2)}$$

Since wages for a man are Rs. 100 and for a woman Rs. 90 per day.

$$\therefore \text{Total wages} = 100x + 90y \qquad \ldots\ldots\ldots \textbf{(3)}$$

Formulation to minimize hiring charges,
Minimize, Z = 100x + 90y.
Subject to the constraint, $x + y \le 7$, $100x + 200y \ge 900$
$200x + 100y \ge 900$, $x, y \ge 0$

(6) A company manufactures two products P and Q. To stay in business, it must produce at least 50 units of P per month. However, it does not have the facilities to produce more than 200 units of product P per month. It also does not have the facilities to produce more than 150 units of product Q per month, while-total demand does not exceed 300 units per month. The profit is Rs. 60 on each unit of P and Rs. 40 on each unit of Q. Formulate LPP to maximize the profit.

Solution:

Let no. of units of product P produced = x,
and no. of units of product Q produced = y.
Given that the company must produce at least 50 units of P per month.

$\Rightarrow x \geq 50$ .... **(1)**

Also given that the company does not have facilities to produce more than 200 units of P and more than 150 units of Q.

$\Rightarrow x \leq 200$ and $y \leq 150$ .... **(2)**

Also total demand does not exceed 300 units.

$\Rightarrow x + y \leq 300$ .... **(3)**

Since profit per unit is Rs. 60 for P and Rs. 40 for Q.

$\therefore$ Total profit = $60x + 40y$ .... **(4)**

Formulation to maximize profit,
Maximize, $Z = 60x + 40y$.
Subject to the constraint, $x \geq 50, x \leq 200, y \leq 150$
$x + y \leq 300, y \geq 0$.

(7) Geeta wants to buy x oranges and y peaches from the store. She must buy at least 5 oranges and the number of oranges must be less than twice the number of peaches. An orange weights 150 grams and a peach weight 100 grams. Geeta can carry not more than 3.6 kg of fruits home. Oranges cost Rs. 8 each and peaches cost Rs. 15 each. Formulate LPP to maximize cost.

Solution:

No. of oranges bought = x

No. of peaches bought = y.

Given that Geeta must buy at least 5 oranges and the number of oranges must be less than twice the number of peaches.

$\Rightarrow x \geq 5$ and $x \leq 2y$ (i.e.) $x - 2y \leq 0$ .... **(1)**

Also given that an orange weights 150 grams and a peach weights 100 grams, and Geeta can't carry more than 3.6 kg.

$\Rightarrow 150x + 100y \leq 3600$ .... **(2)**

Since oranges cost Rs. 8 each and peaches cost Rs. 15 each.

$\therefore$ Total cost = $8x + 15y$ .... **(3)**

Formulation to maximize cost,

Maximize, $Z = 8x + 15y$.

Subject to the constraints, $x \geq 5$, $x - 2y \leq 0$

$150x + 100y \leq 3600$, $y \geq 0$.